

# **Jazz Line and Augmented Scale Theory:** **Using Intervallic Sets to Unite Three- and Four-Tonic Systems**

by Javier Arau  
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## **INTRODUCTION**

Contemporary jazz music is experiencing a renaissance of sorts, as the tonal language of early jazz and bebop, deeply rooted in Western harmony and still an influential and governing force in current music, increasingly mingles with a chromatic and atonal palette. Rather than embracing a free-for-all, chromatic and “outside” approach introduced to jazz decades ago, many musicians today increasingly seem to be interested in grounding atonal chromaticism within a tonal system. Current trends in chromaticism seem to have been influenced considerably by a philosophy, espoused by artists such as David Liebman<sup>1</sup> and George Garzone,<sup>2</sup> claiming that a strong harmonic line can hold its own when superimposed over any chord change, however unrelated. Liebman suggests providing balance to such chromatic superimposition by making at least an occasional return to tonal “diatonic lyricism,”<sup>3</sup> and he is absolutely correct when he stresses that experience, not to mention a very aware and developed set of ears, can help the jazz musician find a successful chromatic approach.<sup>4</sup> In fact, the challenge for a musician might not lie in working solely within an atonal system; set theory and other methods of manipulating intervals already have proven to be popular in handling such material. It is often in bridging between atonal and tonal landscapes that problems arise. Chromatic approaches to tonal harmony are becoming so prevalent among jazz artists that a player who adds only an occasional flat-9 or flat-13 on a V chord can sound entirely dated.<sup>5</sup> If chromatic notes like the flat-13 have become so common that they are often preferred to tonal choices such as the 5<sup>th</sup> of a V chord, then there is a reason for developing a specific theoretical system that embraces chromaticism as a more structural element of jazz harmony, reconciling “inside” and “outside” harmonic development and aiding any musician in the quest to develop a chromatic language within a broader tonal context.

A clear solution to the challenges of moving freely between chromaticism and tonality lies with the augmented scale, a symmetrical pitch collection that successfully unites tonal and atonal systems. This paper, divided into two parts, examines how the augmented scale acts as such a unifying agent. The first part explores the relationship between the 3-tonic system embedded within the augmented scale and the 4-tonic system relating to classic jazz and tonal harmony, surveying how chromatic interval sets within the augmented scale imply harmonies embedded within both systems. The first part also stresses, in particular, the significance of the flatted-6<sup>th</sup> as the primary bridge between these two systems. The second part introduces a complete chromatic palette structured around the four augmented scales, revealing unifying harmonic and geometrical relationships between them and traditional tonic-dominant harmonies, helping to explain how certain harmonies may be superimposed over one another. What begins, in part

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<sup>1</sup> David Liebman, A Chromatic Approach to Jazz Harmony and Melody. (Rottenburg, Germany: Advance, 1991)

<sup>2</sup> George Garzone, personal interview, Sept. 1998.

<sup>3</sup> Liebman, 25.

<sup>4</sup> Liebman, 14.

<sup>5</sup> This is only an observation and is not intended to be a value judgment made towards the artist making such a choice.

one, as a study of the flatted-6<sup>th</sup> as a common entry point into chromaticism, evolves, in part two, to reveal a complete chromatic system embracing the entire tonal and atonal spectrum. While this paper focuses on jazz line and augmented scale theory, augmented scale theory also has broad implications for use as an aid in theoretical analysis and in musical composition.

## PART ONE – The augmented scale and 3- and 4- tonic systems

### 1.1 A common chromatic landscape: The augmented scale and the 3-tonic system

Principles of set theory can be valuable when generating and developing atonal music.<sup>6</sup> Two common intervallic pitch sets that continue to be prevalent in modern music are the (013) and (014) trichords (see Fig. 1). The (014) is of particular interest here, as it alone generates the augmented scale (Fig. 2), a hexatonic scale comprised of alternating minor second and minor third intervals.

Fig. 1: (013) and (014) trichords

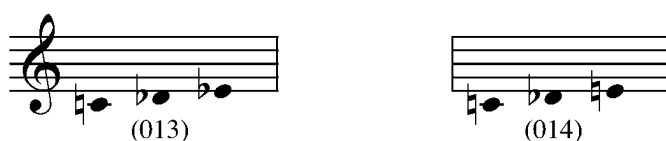
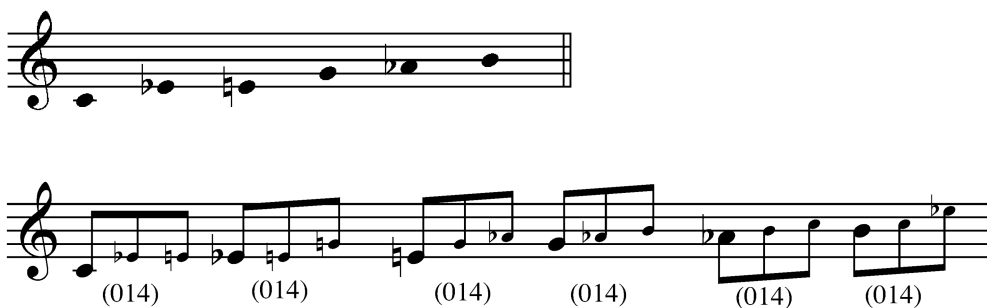


Fig. 2: C augmented scale with embedded (014) trichords



Central to tonal harmony is the tonic-dominant relationship, but an augmented scale is commonly ordered in a way that helps outline its implicit augmented chords (Fig. 3). It also highlights the scale's tendency toward tonal ambiguity, as a clear hierarchical tonal relationship between augmented chords does not exist.

<sup>6</sup> In set theory, the twelve chromatic pitches are each assigned a number, beginning with zero (0). These numbers then help generate pitch sets which allow for quick and easy modulation and development and can help give the work an organic structure. The sets are numbered from smallest interval to largest, regardless of their order. Thus, the pitch set C-Eb-E is considered an (014) trichord, not an (034). For a more involved introduction to set theory, consult Allen Forte's [The Structure of Atonal Music](#) or a similar text.

Fig. 3: C augmented scale, highlighting alternating B+ and C+ chords



Deemphasizing the A $\flat$  and highlighting the G in the C augmented scale below (Fig. 4) helps illustrate how the augmented scale can hint at a tonal hierarchy, where C major is the tonic I chord and G and B imply a V chord. Now the A $\flat$  can be seen as an upper neighbor with a tendency to resolve to the G. The same can be said for B and E $\flat$ , as they tend to move to C and E, respectively.

Fig. 4: C augmented scale, highlighting embedded C major tonality



Embedded within the augmented scale are three major triads and three minor triads (Fig. 5). These triads help form a 3-tonic system, most commonly identified with John Coltrane's "Giant Steps" and also explored in countless other compositions in jazz (see Fig. 6 for a sample chord progression). Coltrane's chromatic approach to composition and improvisation continues to influence modern jazz harmony and is integral to understanding augmented scale theory. A 3-tonic system can be viewed geometrically as an equilateral triangle. Its three points represent the three major chord "tonics" (I) and its sides represent their corresponding, implied dominant (V7) harmonies (Fig. 7).

Fig. 5: Embedded major/minor triads within the C augmented scale

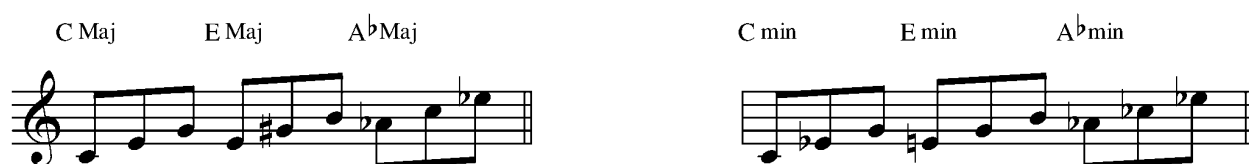


Fig. 6: "Coltrane Changes" superimposed over a II – V – I progression (key of C)

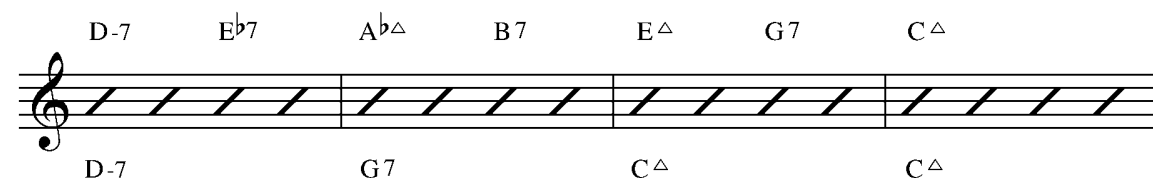
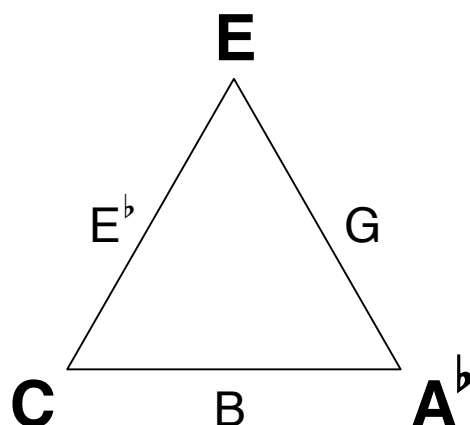


Fig. 7: 3-tonic system geometrically represented by an equilateral triangle.



## 1.2 Classic jazz roots: The diminished scale and the 4-tonic system

The 4-tonic system is derived from a fully diminished scale, a symmetrical octatonic scale built upon (013) trichords (Fig. 8). When major triads are built upon scale degrees 1, 3, 5, and 7 of a “whole-half” diminished scale, 4 tonics, separated by minor third intervals may be formed (Fig. 9). II-V chords and II-V chord substitutions, including the tritone substitution and the minor third substitution<sup>7</sup> common to classic jazz, stem from the II-V chords that correspond with each of these four tonics (Fig. 10).<sup>8</sup> These embedded chords make 4-tonic systems essential and relevant to classic jazz and bebop harmony.

Fig. 8: C diminished scale (whole-half), with embedded (013) trichords

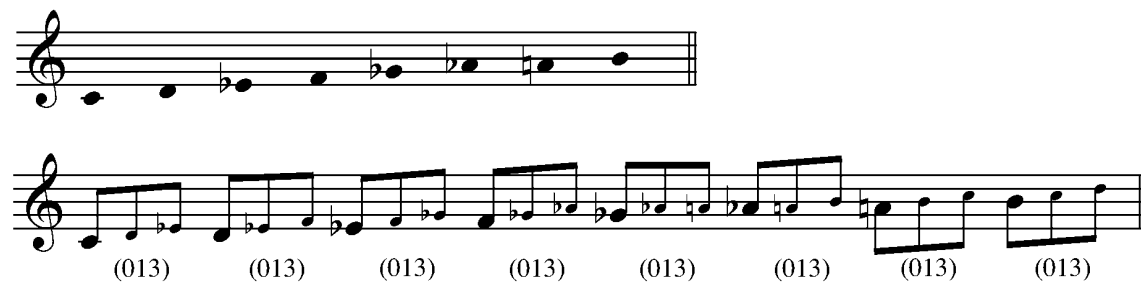
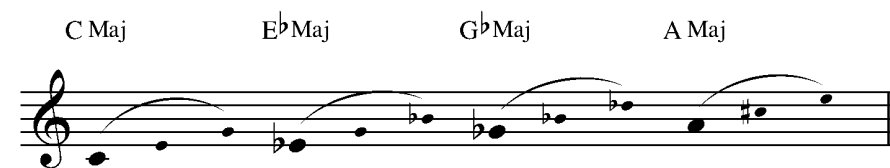


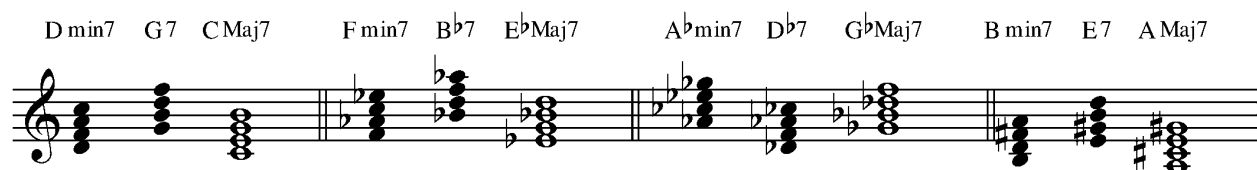
Fig. 9: The four tonics, separated by minor thirds.



<sup>7</sup> Liebman argues that every II-V superimposition has a unique tension-release relationship to the prevailing tonic, dependent largely upon how many common tones the superimposed chord shares with the original. The II-V minor third substitution exhibits a particularly low scale of dissonance because it shares several common tones with the original II-V (p. 19).

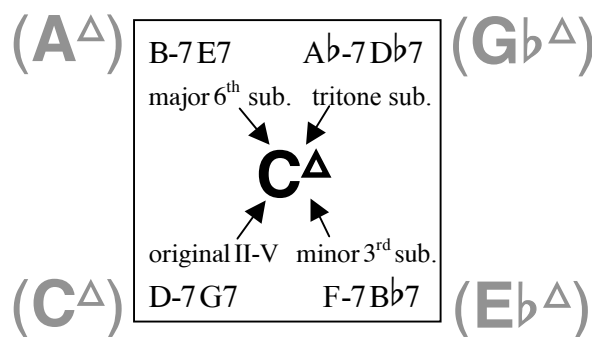
<sup>8</sup> The fourth tonic, A major, often becomes more practical when changed to A minor, thus becoming the relative minor of a C tonic.

Fig. 10: II-V chord progressions stemming from the four tonics



A 4-tonic system can be viewed geometrically as a square—its four points corresponding to the four tonics—but because each II-V substitution traditionally resolves to the same tonal center, a more helpful illustration keeps a singular tonic and positions the four implied II-V harmonies in the square's four corners (Fig. 11).

Fig. 11: 4-tonic square, with singular tonal center and implied II-V chords



### 1.3 Getting from here to there: Using implied leading tones to bridge 3- and 4-tonic systems

Interval sets within the augmented scale naturally imply specific embedded chords. When the semitones in each (014) trichord are treated as leading tones, these interval sets also imply many of the harmonies embedded within the 4-tonic system. In this context, it will be useful to think of the leading tone as both the note lying a semitone below any tonic and also as a raised fourth resolving to the fifth, also known as a double leading tone. That being considered, the pitch collection B-C-E $\flat$  can function as part of a C minor scale or an F minor scale (Fig. 12a). In Figure 12b, pitch collection E $\flat$ -E-G includes flatted-7<sup>th</sup> and raised 7<sup>th</sup> leading tones approaching F minor. F minor is strongly implied when the next note in the augmented scale, an A $\flat$ , is added to form the tetrachord E $\flat$ -E-G-A $\flat$ . The pitch set E-G-A $\flat$  (not shown below) also implies F minor. In Figure 12c, the pitch collection G-A $\flat$ -B can imply an A $\flat$  minor scale. When followed by a C—forming an (0145) tetrachord—it can imply another F minor chord (or a Dm7( $\flat$ 5)). The fact that F minor, in particular, can be so consistently implied is of great significance to bridging between systems.

Fig. 12a-c: Augmented scale sets can imply 4- tonic harmonies

Fig. 12a:



Fig 12b:



Fig 12c:

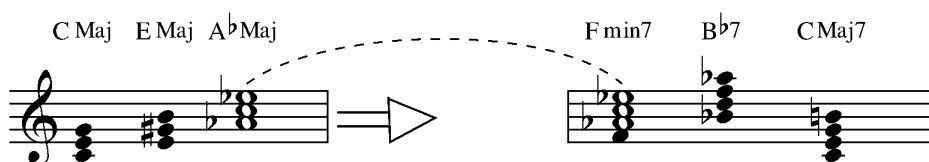


#### 1.4 The gatekeeper: The flatted-6<sup>th</sup> as the structural link between 3- and 4- tonic systems

4-tonic systems, integral to bebop harmony, and 3-tonic systems, integral to more chromatic harmonies stemming from “Coltrane changes,” share some embedded chords that can act as pathways from one tonal palette to the other. Examining these pathways is of import to creative development for practical reasons: the 4-tonic system, particularly because of its implied tritone II-V chord progression, serves as the primary harmonic focus for countless jazz musicians. 3-tonic systems, on the other hand, only tend to be explored when handling works by Coltrane, compositions that even then tend to be performed by only the bravest of players. Augmented scale theory, at the very least, can assist musicians as they move beyond digital patterns and other less than creative approaches when working with 3-tonic systems, helping their improvisations move from tired to inspired.

The flatted-6<sup>th</sup> serves as a pivotal pitch in 3- and 4- tonic systems. In the key of C, A♭ minor is of significance to both systems, functioning as one of the three minor chord pillars of the 3-tonic system and as the tritone substitution of the II chord within the 4-tonic system. A♭ major is equally important. It functions as one of the major tonics of the 3-tonic system, and it is the relative major of an F minor chord, which functions as the minor third substitution of the II chord in the 4-tonic system (Fig. 13).

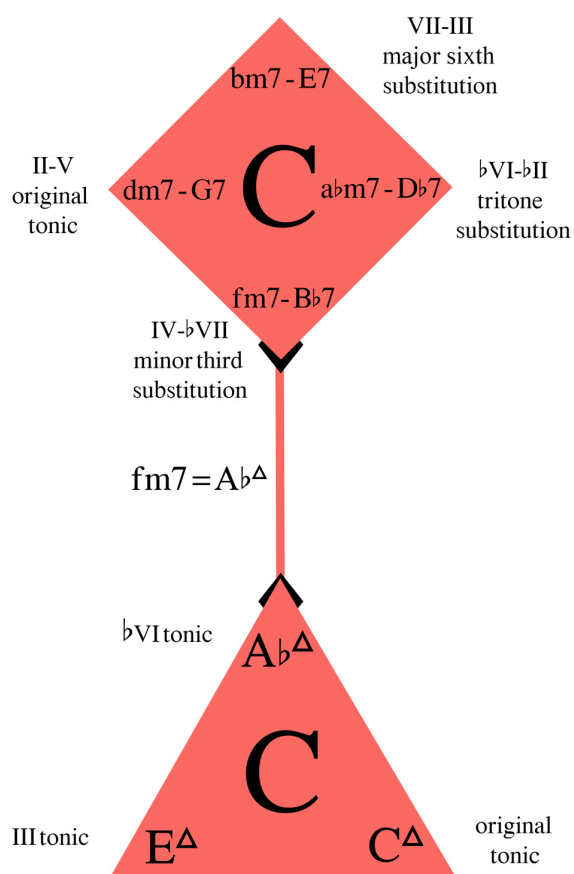
Fig. 13:  $A^b$  major—the link and passageway from a 3-tonic system to a 4-tonic system



The link between 3- and 4- tonic systems can also be viewed geometrically (Fig. 14).

Fig. 14: 4-tonic square and 3-tonic triangle, linked in the key of C by F minor/ $A^b$  major

## 4-tonic system (key of C) with related II-V chords

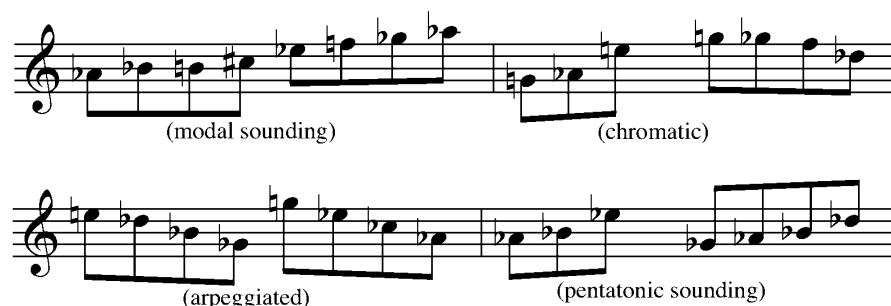


## 1.5 Establishing a chromatic hierarchy: Interval sets unite common chords

Interval sets are structurally significant to augmented scale theory, not only because (013) and (014) trichords form the basis for 3- and 4-tonic systems, but also because they can be used effectively in practical application to unite harmonies possessing varying levels of tension. Scales and modes, by design, are also built from interval sets, and, on one level, sets represent a convenient way of labeling any small melodic or harmonic fragment. Whether sets and intervallic improvisation remain integral to the composer or improviser is up to the musician, who might prefer to develop harmony and melody by exchanging the terminology of sets with a similar concept of scale fragments.

Scale fragments date back to the Greater Perfect System, the ancient Greek root of Western harmony, which was built upon the concept of tetrachords.<sup>9</sup> The D Dorian minor scale, for example, is built upon two like tetrachords separated by a whole step: D-E-F-G, A-B-C-D. This is significant for improvisers and composers alike, who often choose to explore only a portion of a scale before moving to another scale entirely. A small collection of pitches often forms a scale fragment that belongs to more than one scale. For instance, the pitch collection D-E-F forms an (013) trichord, and it also represents a fragment from scales belonging to D minor, G dorian minor, E7(b9) and several other scales and chords. Whether viewed as interval sets or scale fragments, augmented scale theory maintains a view that all pitch collections hold a hierarchical placement in the chromatic spectrum, governed by the gravity of both underlying and implied harmonies. For instance, in the key of C, A<sup>b</sup> major and A<sup>b</sup> minor are structural chords in the augmented scale theory hierarchy. In the example below (Fig. 15), the chromatic chords C<sup>#</sup> minor, E<sup>b</sup> minor, and B<sup>b</sup> minor, can be viewed as belonging to A<sup>b</sup> major and A<sup>b</sup> minor through shared scale fragments or interval sets.

Fig. 15: Common interval sets unite C<sup>#</sup>m, E<sup>b</sup>m, & B<sup>b</sup>m with hierarchical chords A<sup>b</sup>Δ and A<sup>b</sup>m.  
(These sample 3 and 4 note sets can be heard as relating to any of the above chords.)



What matters is not what exact scale or mode is used—for instance, whether, over a G7 chord, a musician plays B<sup>b</sup> minor or F minor (both of which stem from augmented scale theory's structural A<sup>b</sup> harmony—but, rather, how each note relates to the chord tones that surround it. In his book, *A Chromatic Approach to Jazz Harmony and Melody*, Liebman analyzes the first two beats of the line below as a B<sup>b</sup>min7 over a G7 chord, but it just as accurately could be analyzed as an Fmin7 superimposition (Fig. 16).<sup>10</sup>

<sup>9</sup> "Greece," *The New Harvard Dictionary of Music*, 1986.

<sup>10</sup> Liebman, 21.

Fig. 16: The first four notes of this line outline B $\flat$ min7 or Fmin7



Because of the tendency for F and A $\flat$  to resolve to E and G, respectively, the F minor chord also takes on characteristics of a dominant G7 chord. Whether a musician, when playing a line over G7 (key of C) (Fig. 17), thinks of his line as stemming from an F Lydian diminished scale, or, perhaps, an F Dorian minor scale with a double leading tone, has far less musical relevance than how the particular scale fragments resolve (or relate) to given and implied harmonies.

Fig. 17: This line could be derived from F Lyd. dim., F Dorian minor (with leading tones), or G mixolydian (add  $\flat 9$ ).



In the key of C, the C augmented scale, or “tonic augmented scale,” proves to support chromatic harmonies involving a generally altered dominant sound palette, and as this palette is so ubiquitous in jazz, the scale serves a broad purpose. However, by limiting augmented scale theory to only one augmented scale, several notes significant to jazz—including (in the key of C) D, F, A, and B $\flat$ —can only be implied pitches, as they do not belong to the original augmented scale. This fact necessitates the introduction of other augmented scales.

## PART TWO – Using four augmented scales to build the chromatic spectrum

### 2.1 The chromatic palette: Tonic, subdominant, dominant, and subtonic augmented scales

Once the subdominant augmented scale (based on F, assuming the key of C), the dominant augmented scale (based on G) and the subtonic augmented scale (based on B $\flat$ ) are introduced into augmented scale theory, all pitches critical to chromatic and tonal harmony can be addressed and examined without relying on any implied notes. The connections between chords become so vast that any interval set can be justified when played over the tonic key. The goal of systematizing a freely chromatic concept of “any note, any time, anywhere” has been reached, and, significantly, at any moment a return to a tonal sound palette can be achieved with much greater ease. In Figures 18a-c, subdominant (F), dominant (G), and subtonic (B $\flat$ ) augmented scales have been illustrated, highlighting the scales’ defining chords.

Fig. 18a: Subdominant augmented scale (highlighting F major) (key of C)



Fig. 18b: Dominant augmented scale (highlighting G major) (key of C)

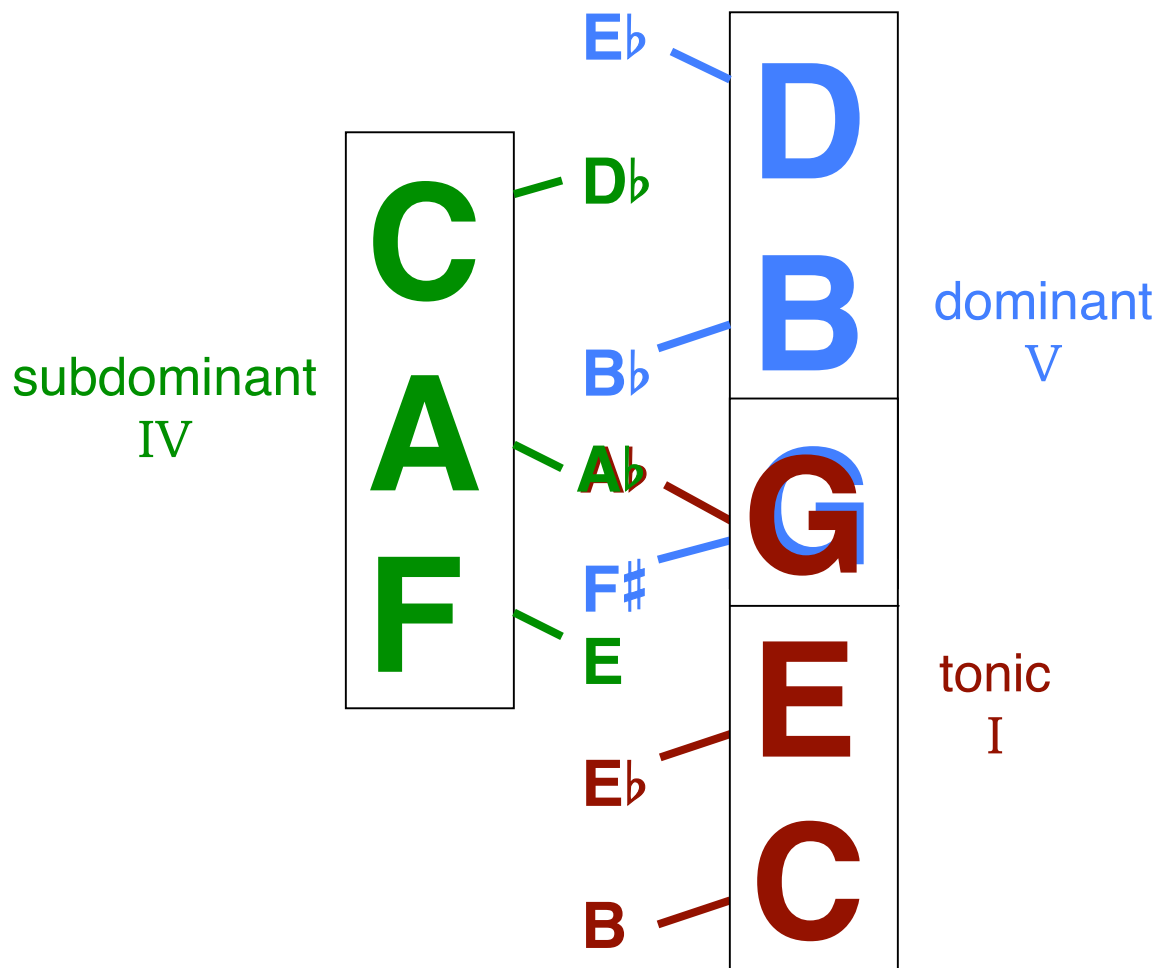


Fig. 18c: Subtonic augmented scale (highlighting Bb major) (key of C)



Figure 19 includes a collection of tonic, subdominant, and dominant augmented scales in the key of C. This ordering helps illustrate how each chromatic pitch in these scales exhibits a tendency to resolve up or down to neighboring chord tones.

Fig. 19: Combined tonic, subdominant and dominant augmented scales



## 2.2 The big picture: The geometry of augmented scale theory

What follows is a series of geometrical illustrations that helps to highlight the logic behind the four augmented scales and how they relate to a corresponding 4-tonic system. The geometry of augmented scale theory contains fractal properties; each illustration functions as a structural component of the illustration that follows. There are too many threads connecting the harmonies in these illustrations to begin to address them at length in this paper, although a few of the most significant connections in each illustration are labeled as primary or secondary pathways. Note that the II-V progressions at the corners of each 4-tonic system are represented only by their corresponding II chords. The V chords have been omitted because they were implied by the existing II chords and thus were deemed redundant.

Figure 20 introduces all four augmented scales and illustrates how they relate to a central tonic. All four augmented scales are structurally integral to augmented scale theory's complete chromatic palette, as evidenced by the geometry in the examples below, but, of the four scales, the subtonic augmented scale is the most chromatic and tonally distant in relation to the central tonic. It is no coincidence that it forms a primary passageway with the most distant and impractical of the 4-tonic system's II-V chord sets, the major 6<sup>th</sup> II-V substitution. Subdominant and dominant harmonies have more structural significance than subtonic harmony in tonal theory. This holds true with augmented scale theory, as well.

Fig. 20: 3-tonic square, annotated

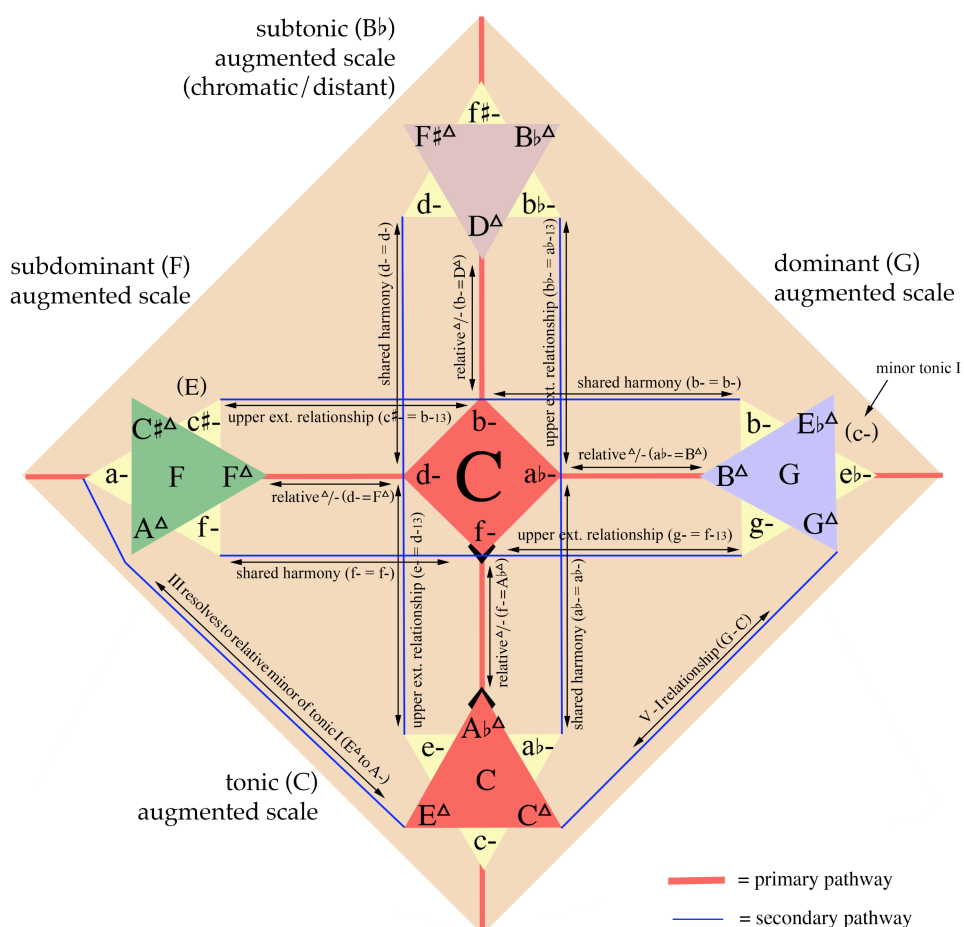


Fig. 21: 3-tonic branch (detail of Complete Chromatic Circle of Tonics)

Note: The central tonic (red triangles and red squares) of each 3-tonic square extends out from the center of this 3-tonic branch.

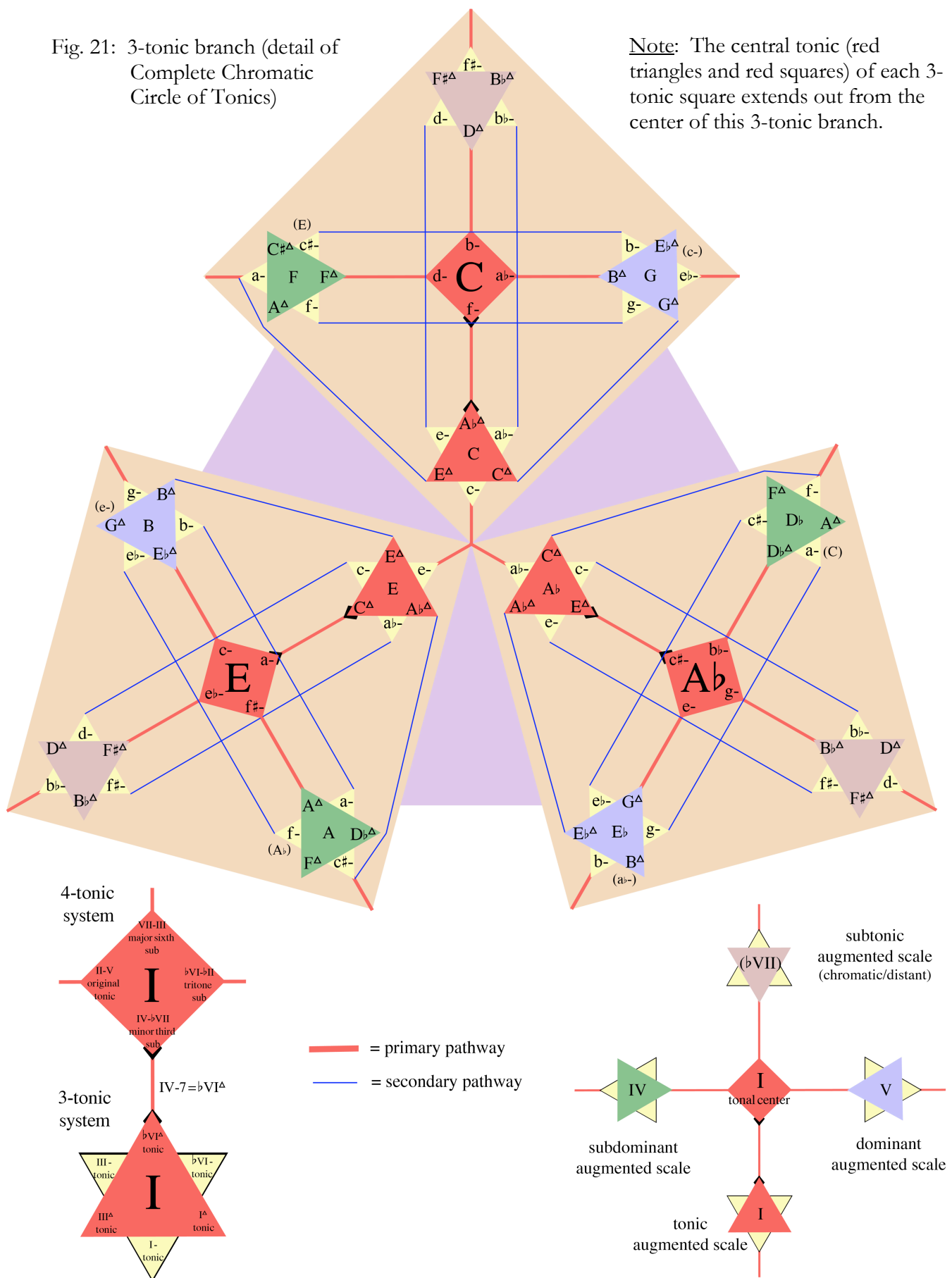
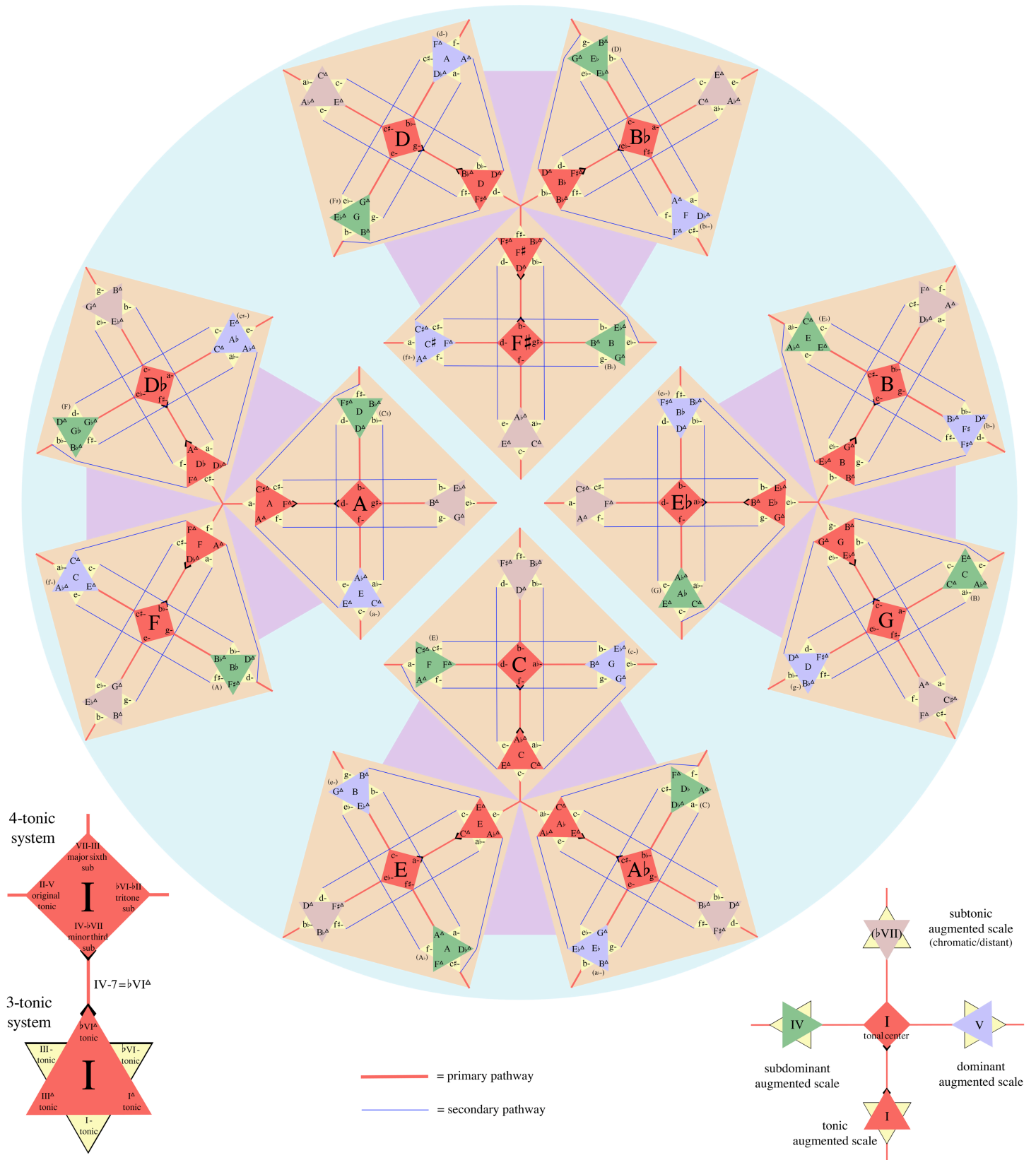


Fig. 22: Augmented scale theory's Complete Chromatic Circle of Tonics



## CONCLUSION

Augmented scale theory holds broad implications for use in musical analysis, but, perhaps more importantly, it is meant also for practical application, helping to ease the steep learning curve that improvisers and composers face in the quest to work with chromatic palettes. A significant advantage to using augmented scale theory is that it distills harmony down to its essential function of tension-release. With this simplification, the musician need not become entangled in long lists of scale options for every chord and can instead dedicate more effort toward controlling a tonal (or atonal) landscape. In addition, when working with the augmented scale as a central tonal structure, the jazz musician can more quickly develop fluency in multiple key centers. Instead of working toward mastery of 12 independent keys, the musician can trim the amount of work down to developing fluency within 4 independent tonic augmented scales.

This paper serves only as a brief introduction to the topic of augmented scale theory. The focus here has been more on jazz line and improvisation than on composition. A more exhaustive study on the topic must include a careful examination of practical applications in improvisation and composition—including the use of wider interval sets and common pentatonic and octatonic scales—and an extensive analysis of historical examples of chromatics and interval sets in jazz, as viewed through this lens.

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